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Collective excitations built on the 2^+_γ state in ^{168}Er

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Abstract. To examine the collectivity of the previously proposed $K^\pi = 4^+$ $\gamma\gamma$-two-phonon state in ^{168}Er at an excitation energy of 2056 keV and of the first excited $K^\pi = 0^+$ band at 1217 keV (b-band), a Coulomb excitation experiment has been carried out at the Heidelberg-Darmstadt Crystal Ball spectrometer. The B(E2) value connecting the $4^+_{\gamma\gamma}$ state to the γ-band was remeasured to be B(E2, $4^+_{\gamma\gamma} \rightarrow 2^+_\gamma$) = (600 ± 130) e^{2}fm^4, which almost exhausts the harmonic expectation. In addition, the B(E2) values connecting the band head of the lowest lying excited 0^+_b band to the 2^+_g and 2^+_γ states were measured to be B(E2, $2^+_\mathrm{g} \rightarrow 0^+_\mathrm{b}$) = (4.4 ± 0.6) e^{2}fm^4 and B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{b}$) = (30.4 ± 4.6) e^{2}fm^4; the latter is almost a factor of 10 smaller than the B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{g}$) value, which shows that the 0^+_b band head has no significant contribution of the $K^\pi = 0^+$ $\gamma\gamma$-two-phonon state in its wave function.

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1 Introduction

The Collective Nuclear Model [1], developed by Bohr and Mottelson in the 1950s, predicts rotational bands in the excitation spectra of deformed nuclei, which are built on the ground state as well as on various low lying intrinsic excitations, the more collective of these usually being attributed to surface vibrations. For strongly deformed even-even nuclei such as e.g. ^{166,168}Er, two types of quadrupole vibrations are predicted: a so-called β-vibration with $K^\pi = 0^+$ and a γ-vibration with $K^\pi = 2^+$, K being the projection of the nuclear angular momentum onto the intrinsic symmetry axis. Indeed, in many well deformed even-even nuclei rotational bands have been found in addition to the ground-state band (g-band), with band heads at energies between 500 keV and 1 MeV, i. e. well below the pairing gap, and spins and parities of 0^+ and 2^+. As their vibrational origin should be reflected by enhanced E2 transition matrix elements between these states and the g-band, the enhancement being expected as a result of the coherent motion of the nucleons involved in the vibration, much experimental effort has been put into measuring the corresponding B(E2) values. The fact that the out-of-band transition strengths for most of the lowest $K^\pi = 2^+$ bands are larger than can be accounted for from single particle excitations and their smooth dependence on Z and N usually serves as one of the main arguments in favour of their classification as bands built on a single γ-vibrational excitation. However, for such an interpretation to be more than just a pictorial name the existence of states involving the excitation of more than one γ-vibrational phonon plays a key role. The question whether the simplest of these states, the $\gamma\gamma$-two-phonon states with $K^\pi = 0^+$ and 4^+ exist or not, and if so, to what degree the harmonic approximation can be retained, has thus been a major issue of nuclear structure physics for many years.

For a proper assignment of excited states to two-phonon states the knowledge of absolute out-of-band E2 transition strengths is mandatory. For strongly deformed nuclei only a few examples exist, where measured out-of-band B(E2) values support the interpretation of a $K^\pi = 4^+$ band as a rotational band built upon a $\gamma\gamma$-two-phonon $K^\pi = 4^+$ state:
In the actinide region, Korten et al. [2,3] established a $K^\pi = 4^+$ band with a band head at 1414 keV in ^{232}Th using Coulomb excitation. The band exhibits all features expected for a *harmonic* $\gamma\gamma$-two-phonon band, namely a band head energy of roughly $E(4^+_{\gamma\gamma}) \approx 2 \times E(2^+_\gamma)$ and a B(E2) value for its decay to the one-phonon γ-vibrational state of approximately B(E2, $4^+_{\gamma\gamma} \rightarrow 2^+_\gamma$) $\approx 25/9 \times$ B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{g}$).
In the rare earth region, three cases are documented so far: In ^{168}Er, from a lifetime determination of the $I^\pi = K^\pi = 4^+$ state at 2056 keV, using the ^{167}Er(n,γ)^{168}Er reaction together with the GRID technique, Börner et

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al. [4] extracted a B(E2) value of $B(E2, 4^+_{\gamma\gamma} \to 2^+_\gamma) \approx (1.1 \pm 0.6) \times B(E2, 2^+_\gamma \to 0^+_g)$, reflecting $\approx (40 \pm 20)$ % of the harmonic value. In ^{166}Er, Fahlander et al. [5,6] observed two 4^+ states in a Coulomb excitation experiment, which together exhaust again $\approx$ 40 % of the harmonic $\gamma\gamma$ strength, a result recently confirmed in (n,n′γ) studies [7]. Moreover, evidence for a $K^\pi = 4^+$ state in ^{164}Dy, exhausting $\approx$ 35 % of the harmonic $\gamma\gamma$ strength has been presented by Corminboeuf et al. [8]. Together with the corresponding excitation energies of $E(4^+)/E(2^+_\gamma) \approx 2.5$ for the two Er-isotopes and ≈ 2.85 for ^{164}Dy these findings are taken as a verification of an anharmonic $\gamma\gamma$-two-phonon vibration. Indeed, due to the slightly γ-deformed equilibrium shape of these otherwise strongly β-deformed nuclei anharmonicity effects are expected, which result in an increase of the $E(4^+_{\gamma\gamma})/E(2^+_\gamma)$ ratio and in a decrease of the B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value as compared to the harmonic prediction [9,10].

While our data basis on $K^\pi = 4^+$ two-phonon states in deformed nuclei is still rather scanty, the situation is even worse as far as our experimental knowledge of the expected $K^\pi = 0^+$ $\gamma\gamma$-two-phonon states in these nuclei is concerned. In ^{166}Er Fahlander et al. [5,6] observed in their Coulomb excitation work a state at 1943 keV and suggested this to be a candidate for the $K^\pi = 0^+$ $\gamma\gamma$-two-phonon state on the basis of its excitation energy being close to the $K^\pi = 4^+$ $\gamma\gamma$-two-phonon state and of its enhanced E2 transition to the γ-band of about twice the γ- to g-band transition. This conjecture was recently corroborated by Garret et al. [7]. For ^{168}Er as well as for other deformed rare earth nuclei, however, experimental and theoretical arguments [11–13] have been brought forward that the lowest excited $K^\pi = 0^+$ state is carrying a significant fraction of the $\gamma\gamma$ strength. The experimental arguments are mainly based on the sizable decay branches of the higher spin members of these $K^\pi = 0^+$ bands to the γ-band. In ^{168}Er, in particular, where the $K^\pi = 0^+$ band head is located at 1217 keV, the branching ratios of the higher spin members of this band to the g- and γ-band are known due to the (n,γ) work of Davidson et al. [14, 15]. As pointed out already by Günther et al. [16], however, the enhanced decays of the higher spin states to the γ-band can as well be explained by small $\Delta K = 2$ admixtures of the γ-band into the excited $K^\pi = 0^+$ band; direct measurements of the E2 transition strength between the $K^\pi = 0^+$ band head and the γ-band are necessary to settle this issue.

In the present experiment we used the same experimental technique we employed already in our investigations on ^{232}Th [2] and ^{166}Er [5] to search for collective excitations built on the γ-band in ^{168}Er. Our method, which is based on the Coulomb excitation process with heavy projectiles in conjunction with a 4π γ-array to suppress high γ-multiplicity events, is well suited for this purpose and allows to determine absolute B(E2) values. The main aim of the experiment was to remeasure the rather unprecisely known B(E2) value between the proposed $4^+_{\gamma\gamma}$ state and the 2^+_γ state and to determine the absolute E2 strength between the lowest excited $K^\pi = 0^+$ state and the g- and γ-band. The results will be used to critically examine the relevance and usefulness of the vibrational picture in strongly deformed nuclei.

2 Experimental method and data analysis

The experiment was performed at the Heidelberg-Darmstadt Crystal Ball spectrometer [17]. A 1.2 mg/cm^2 target enriched to 95.5 % in ^{168}Er was bombarded by a pulsed (pulse distance 74 ns) ^{58}Ni beam of 225 MeV delivered by the accelerator facility at the MPI für Kernphysik in Heidelberg. This beam energy was chosen such that the distance between the nuclear surfaces during the collision process was always larger than 5 fm, assuring a purely electromagnetic interaction between the two colliding nuclei, an important prerequisite if B(E2) values are to be extracted. Scattered projectiles were detected by an eightfold-segmented annular silicon strip detector covering the laboratory scattering angles 116° to 134°. The Er-nuclei recoil with an average velocity of ≈0.04c into vacuum and are finally stopped in a Pb-layer 10 mm behind the target. The γ-ray detection setup consisted of the Crystal Ball together with a Ge-detector placed under 0° with respect to the beam axis at a target-detector distance of 93 mm. Only the particle- and the Ge-detector were required in coincidence as trigger condition for the data acquisition.

Figure 1 shows the relevant decay branches of the states under investigation, as measured by Davidson et al. [14,15]. The $K^\pi = 4^+$-rotational band at 2056 keV is denoted by "$\gamma\gamma$" and the lowest excited $K^\pi = 0^+$ band by "b". In addition to the expected ($\gamma\gamma \to \gamma \to$ g) decay mode, transitions involving an isomeric ("Iso") $K^\pi = 4^-$

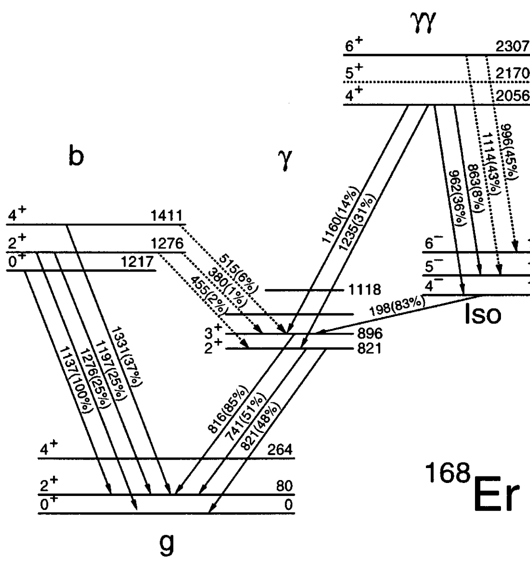

Fig. 1. Partial level scheme of ^{168}Er, showing the relevant decay modes of the proposed $\gamma\gamma$-band and of the 1217 keV 0^+ band (b-band) [14,15], measured directly (solid lines) and indirectly (dotted lines) in the present work

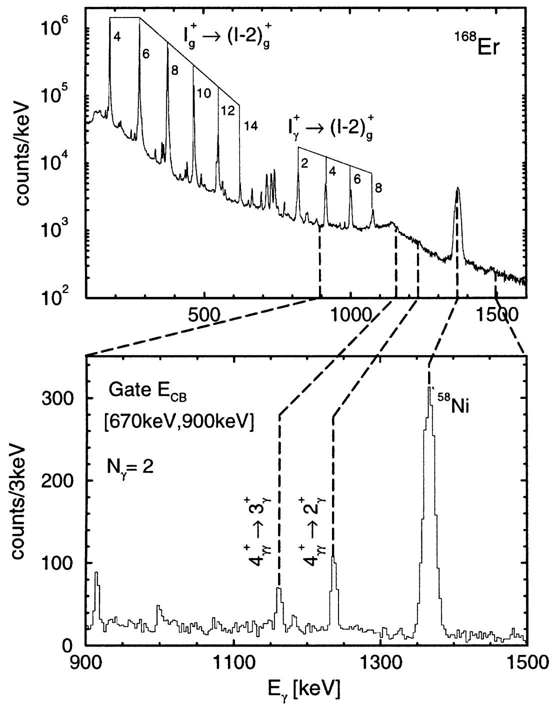

Fig. 2. Doppler corrected Ge-spectra for Coulomb excitation of ^{168}Er with ^{58}Ni at a beam energy of 225 MeV. Upper part: Total Ge-spectrum without conditions; lower part: Part of the coincident Ge-spectrum with the condition that exactly one γ-ray was detected in the Crystal Ball with an energy corresponding to the decays of the lowest two states of the γ-band

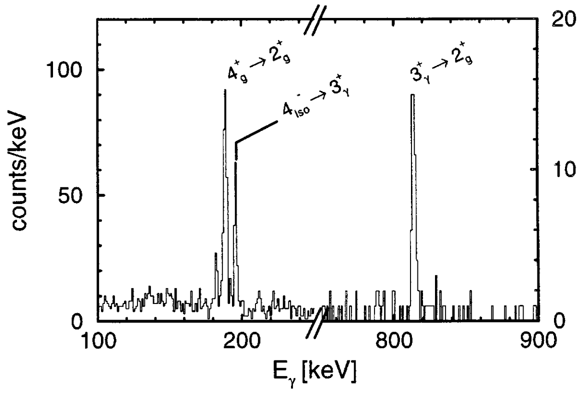

Fig. 3. Ge-spectrum of the delayed decay branch of the $K^\pi = 4_{\gamma\gamma}^+$ band via the isomeric 4^- state; the conditions imposed are given in the text. No Doppler correction was applied as the delayed decay occur from Er-nuclei at rest in the Pb-stopper. The $4_g^+ \to 2_g^+$ events are accidentally

band at 1094 keV are observed. While ($4_{\gamma\gamma}^+ \to \gamma \to$ g) events are characterized by a prompt γ-multiplicity of $N_\gamma = 2$, events stemming from the ($4_{\gamma\gamma}^+ \to$ Iso $\to \gamma \to$ g) decay have $N_\gamma = 3$, the first photon being emitted prompt and the two following transitions being considerably delayed due to the long lifetime of the $K^\pi = 4^-$ state of 157 ns. Of the (b $\to$ g) transitions only those having $N_\gamma = 1$ are shown in Fig. 1. Note, that the (anyhow highly converted) $2_g^+ \to 0_g^+$ or $5^- \to 4^-$ transitions are removed by an energy cutoff in the analysis and therefore cannot contribute to the γ-multiplicity. The chosen combination of the Crystal Ball with its outstanding detection efficiency and a high resolution Ge-detector is perfectly suited for selecting such low multiplicity events.

The upper part of Fig. 2 shows as an example a Ge-spectrum without any conditions imposed. In the lower part, only those events were kept where, in addition to the photon in the Ge-detector, exactly one photon was detected in the Crystal Ball (CB) within the energy range of $E_{\rm CB} \in$ [670 keV, 900 keV] covering the decays of the lowest two states of the γ-band to the g-band. This condition leads to a very good suppression of uninteresting events and results in a clear signal for ($4_{\gamma\gamma} \to I_\gamma$) decays.

Decays involving the isomeric 4^- state ($\tau = 157$ ns) are analyzed by requiring the detection of one prompt photon in the Crystal Ball in the energy range $E_{\rm CB} \in$ [700 keV, 1100 keV], corresponding to the ($\gamma\gamma \to$ Iso) decays, and one delayed photon with an energy of either $E_{\rm CB} \in$ [120 keV, 280 keV] or $E_{\rm CB} \in$ [550 keV, 1050 keV] corresponding to the (Iso $\to \gamma$) or ($\gamma \to$ g) transitions, respectively. In the resulting Ge-spectrum, obtained by allowing for delayed Ge-events up to 40 ns (larger delay times were not accessible due to the timing of the hardware trigger), the $4_{\rm Iso}^- \to 3_\gamma^+$ and $3_\gamma^+ \to 2_g^+$ transitions are clearly visible (Fig. 3). Background and prompt cascades are suppressed by four orders of magnitude, as estimated by comparing the background and the remaining accidental $4_g^+ \to 2_g^+$ transitions in the Ge-spectrum to those of the total Ge-spectrum in the upper part of Fig. 2. Note that the accidental γ-rays from the $4_g^+ \to 2_g^+$ transition in Fig. 3 are fully Doppler shifted as no Doppler shift correction was applied to the γ-spectrum. The prompt decay branch of the $6_{\gamma\gamma}^+$ state to the γ-band is only about 12 % and was not observed in this experiment. However, since the $4_{\rm Iso}^-$ state is fed by both the $4_{\gamma\gamma}^+$ and $6_{\gamma\gamma}^+$ states, the excitation probability of the $6_{\gamma\gamma}^+$ state can be estimated from the observed intensity in the delayed $4_{\rm Iso}^- \to 3_\gamma^+$ transition. The $5_{\gamma\gamma}^+$ state does not contribute as it has unnatural parity and is therefore only weakly excited ($P(I_{\gamma\gamma}^{\rm odd})/P(I_{\gamma\gamma}^{\rm even}) < 0.1$). A comparison of the $4_{\rm Iso}^- \to 3_\gamma^+$ intensity to the one deduced from the prompt $4_{\gamma\gamma}^+ \to \gamma \to$ g cascades, yields P($6_{\gamma\gamma}^+$) / P($4_{\gamma\gamma}^+$) ≈ 0.5, which is in agreement with Coulomb excitation calculations (see below).

Direct transitions from the b- to the g-band were examined by demanding $N_\gamma = 1$, the single photon being detected in the Ge-detector. If one requires no hit in the Crystal Ball ($N_{\rm CB}$=0), the spectrometer works not only as a multiplicity filter but also as a highly efficient Anti-Compton shield for the Ge-detector. The relevant region of the resulting Ge-spectrum is shown in Fig. 4. The positions of the expected transitions from the b-band to the g-band are indicated. The decay branch from the b-band to the γ-band, which in principle can be selected for events having a γ-multiplicity of $N_\gamma = 2$ in the same way as the $4_{\gamma\gamma}^+ \to I_\gamma^+$ transitions (see Fig. 1) could not be observed

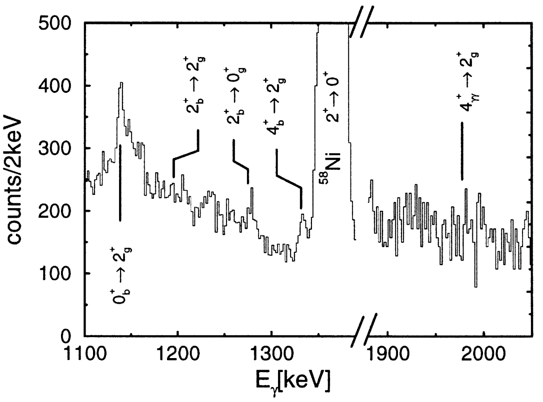

Fig. 4. Doppler corrected Ge-spectrum under the condition $N_{\rm CB} = 0$. The position of the expected (b → g) decays as well as of the $4^+_{\gamma\gamma} \to 2^+_\gamma$ transition are indicated

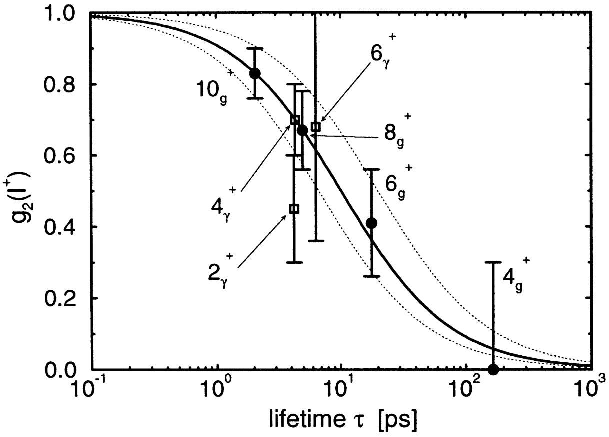

Fig. 5. Measured deorientation coefficients as a function of the lifetime of the individual states. They are compared to an Abragam-Pound type parametrization (smooth lines)

in the present experiment due to the weak population of the b-band.

Direct transitions from other excited $K^\pi = 0^+$ bands to the g-band were not observed. Taking into account our observational limit and assuming a direct population of that band from the g-band in a one-step Coulomb excitation process, an upper limit of B(E2, $2^+(K^\pi = 0^+) \to 0^+_{\rm g}) < 10$ e^{2}fm^4 can be estimated, which confirms the lack of a classical β-vibrational band in this nucleus up to an excitation energy of 2 MeV.

3 Analysis and results

The experimental excitation probabilities for the $4^+_{\gamma\gamma}$ state as well as for the members of the b-band are extracted from the different $\gamma_{\rm CB} - \gamma_{\rm Ge}$ coincidence data shown in the previous figures. The observed intensity is mainly caused by the direct population in the Coulomb excitation process. A possible feeding from above is estimated within the rigid rotor picture.

As one of the deexcitating γ-rays is detected at $\langle\vartheta\rangle = 0°$ in the Ge-detector, one has to take into account the angular distribution of these γ-rays. It depends on the spins of the involved states, the mixing ratios of the transitions, the alignment of the initial state, and the degree of deorientation due to the perturbation of the angular distribution caused by the hyperfine interaction of the atomic magnetic field with the nucleus recoiling into vacuum. The alignment of the inital state can be reliably obtained from Coulomb excitation calculations (see below), while the deorientation is estimated using the parametrisation of Abragam and Pound [18], assuming an exponential attenuation of the alignment of the individual nuclear states from the instant of population up to the instant of decay. In the present geometry this effect causes an attenuation of the angular correlation $\sum a_K P_K(\cos\vartheta)$, which can be described by multiplying the unperturbed angular correlation coefficients a_K by the attenuation factors G_K given by

$$G_K(I_i) = \prod_j g_K(I_j)$$
$$\text{with } g_K(I_j) = (1 + \tfrac{1}{3}K(K+1)\Lambda\,\tau(I_j))^{-1}. \qquad (1)$$

For more details see [19]. The index j runs over all states from the one initially populated up to the state I_i, from which the decay is observed. The parameter Λ depends on the nuclear g-factor and is expected to be constant at least for states belonging to the same band.

In order to determine the parameter Λ and to show the reliability of this parametrisation, we determined the degree of deorientation for some members of the g- and γ-band from angular distributions measured in the Crystal Ball. The extracted deorientation coefficients g_2, which are shown in Fig. 5 as a function of lifetime, are well reproduced using $\Lambda = 0.050$ ps^{-1} (solid line) ± 0.025ps^{-1} (dashed lines). Since the intrinsic structure and thus the g-factors of the $4^+_{\gamma\gamma}$ and $4^+_{\rm b}$ states are expected to be very similar to those of the g- or γ-band, the parametrization is used to estimate the deorientation coefficients for these states as well. For the $4^+_{\gamma\gamma}$ state, with $\tau(4^+_{\gamma\gamma})\Lambda \ll 1$, the deorientation effect plays only a minor role and a deorientation coefficient of $g_2 = 0.94 \pm 0.03$ is estimated. For the $4^+_{\rm b} \to 2^+_{\rm g}$ transition, however, which is used to determine the excitation probability of the $4^+_{\rm b}$ state, a large attenuation of the angular distribution has to be taken into account as the lifetime of the $4^+_{\rm b}$ state is estimated to be in the order of 40 ps [19].

The resulting experimental excitation probabilities, normalized to the measured excitation probability of the 2^+_γ state, $R(I_i) = P(I_i)\ /\ P(2^+_\gamma)$, are

$$\begin{array}{lcl} R(4^+_{\gamma\gamma}) & = & (2.1 \pm 0.3) \times 10^{-2} \\ R(0^+_{\rm b}) & = & (1.44 \pm 0.23) \times 10^{-2} \\ R(2^+_{\rm b}) & \leq & 0.3 \times 10^{-2} \\ R(4^+_{\rm b}) & = & (0.90 \pm 0.15) \times 10^{-2}. \end{array}$$

The errors include statistical as well as systematic uncertainties. Note that a large fraction of the error of $R(4^+_{\gamma\gamma})$ originates from the uncertainty in the branching ratio $\Gamma(4^+_{\gamma\gamma} \to 2^+_\gamma)\ /\ \Gamma(\text{tot}) = 0.31 \pm 0.04$ [14,20]. The aim of the analysis is to fit a set of $E\lambda$ matrix elements to these probabilities, which are calculated within the framework of semiclassical Coulomb excitation theory [21], using a standard code (COULEX) [22]. The input to COULEX consists of the kinematic parameters, the level scheme of the investigated nucleus and the reduced $E\lambda$ matrix elements between these states. In the present analysis we included all known states of the g-band (up to $I^\pi = 18^+$), the γ-band (up to 10^+), the b-band (up to 6^+) and the $\gamma\gamma$-band (up to 6^+). As the population probability of an individual state may also be affected by the coupling to other levels, which themselves are not observed, the b-band and the $\gamma\gamma$-band were extended up to spin 10^+ assuming a constant moment of inertia. Other nuclear states like the members of the low lying 4^- band shown in Fig. 1 or the members of the collective octupole vibrational bands contribute with less than 1 % to the excitation probabilities of the investigated states and are therefore omitted in the COULEX calculations. Fortunately, due to previous Coulomb excitation studies [23], all relevant diagonal and in-band matrix elements of the g-band and the γ-band, as well as the inter-band matrix elements between them are known in the spin region under investigation; they were kept fixed during the analysis.

Besides these known matrix elements, the excitation of the $4^+_{\gamma\gamma}$ state depends on the in-band matrix elements within the $\gamma\gamma$-band and the inter-band matrix elements connecting the $\gamma\gamma$- to the γ-band. However, only $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|I^+_\gamma\rangle$ ($I^+_\gamma = 2^+$ and 4^+) and $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|4^+_{\gamma\gamma}\rangle$ contribute significantly to the excitation probability of the $4^+_{\gamma\gamma}$ state. The inter-band matrix element that couples to the $I^+_\gamma = 6^+$ state generally is very small due to the smallness of the corresponding Clebsch-Gordan coefficient and therefore plays only a minor role. The diagonal matrix element $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|4^+_{\gamma\gamma}\rangle$ was estimated assuming an intrinsic quadrupole moment of $Q_0^{\gamma\gamma} = 8.0$ eb, which is the mean value deduced from the measured quadrupole moments of the lowest members of the g- and γ-band [23]. All other matrix elements within the $\gamma\gamma$-band, which were calculated assuming this quadrupole moment, can be shown to have only a small influence on the excitation probability of the $4^+_{\gamma\gamma}$ state. For example, by changing the matrix element $\langle 6^+_{\gamma\gamma}\|\mathrm{E2}\|4^+_{\gamma\gamma}\rangle$ by about a factor of two, the excitation probability of the $4^+_{\gamma\gamma}$ state is only affected by ≈ 5 %. The inter-band matrix element $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|4^+_\gamma\rangle$, as well as all other inter-band matrix elements are calculated from $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|2^+_\gamma\rangle$ using the Alaga rule.

The result of a COULEX calculation using these matrix elements is shown by the solid line in Fig. 6, where the measured probability $P(4^+_{\gamma\gamma})/P(2^+_\gamma) = 2.1 \pm 0.3$ is compared to calculated values as a function of B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$). As the inter-band matrix elements are quite sensitive to small K-admixtures, we tested in addition the influence of a ΔK=2 coupling between the γ- and $\gamma\gamma$-band on the extraction of the B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value. In first order and assuming that the intrinsic quadrupole moments Q_0 of the two interacting bands to be equal, the matrix elements are given by the generalized Alaga rule (see (4-210) and (4-211) of [1])

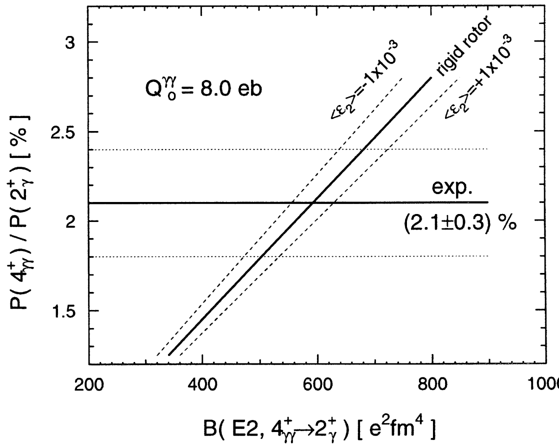

Fig. 6. Results of COULEX calculations for the population ratio $P(4^+_{\gamma\gamma})$ / $P(2^+_\gamma)$

$$\begin{aligned}&\langle K_2 = K_1+2, I_2\|\mathcal{M}(\mathrm{E2})\|K_1, I_1\rangle = \\ &\quad\sqrt{2I_1+1}\langle I_1K_1 22|I_2K_2\rangle \\ &\quad\times (M_1 + M_2(I_2(I_2+1) - I_1(I_1+1))) \\ &\quad\times \begin{cases}\sqrt{2} & K_1 = 0\\ 1 & K_1 \neq 0.\end{cases}\end{aligned} \qquad (2)$$

M_1 and M_2 are quantities which depend on the spin-independent inter-band matrix element $\langle K_2|\mathcal{M}(\mathrm{E2}, \nu = 2)|K_1\rangle$ and on the $\Delta K = 2$ mixing amplitude $\langle K_2|\epsilon_2|K_1\rangle$ through

$$\begin{aligned}M_1 &= \langle K_2 = K_1+2|\mathcal{M}(\mathrm{E2}, \nu=2)|K_1\rangle - 4(K_1+1)M_2\\ M_2 &= \sqrt{\frac{15}{8\pi}}Q_0\langle K_2|\epsilon_2|K_1\rangle.\end{aligned} \qquad (3)$$

From our observed branching ratio I($4^+_{\gamma\gamma} \to 3^+_\gamma$) / I($4^+_{\gamma\gamma} \to 2^+_\gamma$) $= 0.55 \pm 0.10$, an upper limit for the $\Delta K = 2$ mixing amplitude of $|\langle K_{\gamma\gamma} = 4|\epsilon_2|K_\gamma = 2\rangle| < 1 \times 10^{-3}$ can be estimated. The effect of such a mixing in the extraction of the B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value is small as shown by the dashed lines in Fig. 6. This small mixing is indicative of the $K = 4$ purity of the $4^+_{\gamma\gamma}$ band; together with the known mixing amplitude of the γ-band with the g-band of $\approx -8 \times 10^{-4}$ [24] the direct decay to the g-band should be strongly hindered, which is consistent with the small B(E2) ratio of

$$\mathrm{B}(\mathrm{E2}, 4^+_{\gamma\gamma} \to 2^+_{\mathrm{g}})/\mathrm{B}(\mathrm{E2}, 4^+_{\gamma\gamma} \to 2^+_\gamma) < 7 \times 10^{-3}$$

estimated from the upper limit for the ($4^+_{\gamma\gamma} \to 2^+_{\mathrm{g}}$) branch determined from the $M_\gamma = 1$ spectrum (Fig. 4).

Assuming an intrinsic quadrupole moment of $Q_0^{\gamma\gamma} = 8.0$ eb, a value of B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) = (600 ± 110) e^{2}fm^4 can be read off from Fig. 6. The additional assumption of e.g. multi-phonon states, that may couple to the 2^+_γ or the $4^+_{\gamma\gamma}$ state has no significant influence on the extracted B(E2) value. Furthermore, systematic errors in the matrix elements connecting the γ-band to the g-band are eliminated to a great extent in the calculations, as the excitation probability of $P(4^+_{\gamma\gamma})$ is normalized to $P(2^+_\gamma)$. However, a sizable uncertainty of the final B(E2) value arises from the uncertainty of the diagonal matrix element $\langle 4^+_{\gamma\gamma}\|\mathrm{E2}\|4^+_{\gamma\gamma}\rangle$, i. e. of the static quadrupole moment of the $4^+_{\gamma\gamma}$ state, which has a large influence on the excitation probability. A change in the deformation of about 5 %, a typical deviation from the mean value observed for the low spin states in the g- and γ-band [23], changes the extracted B(E2) value by about 10 %. Taking into account this uncertainty, the resulting B(E2) value is

$$\mathrm{B(E2}, 4^+_{\gamma\gamma} \to 2^+_\gamma) = (600 \pm 130)\ \mathrm{e^2fm^4}.$$

While the extraction of the B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value is almost independent of the matrix elements involving higher spin states of the $\gamma\gamma$-band, Coulomb excitation of the b-band looks quite different. The excitation probabilities of individual states of the $K^\pi = 0^+$ band strongly influence each other and a change of individual matrix elements within the $K^\pi = 0^+$ band by as little as 10 % may change the population pattern significantly [21]. Furthermore, two interfering excitation paths, one direct and one via the γ-band have to be considered. Fortunately, for the 6^+ and 4^+ members of the b-band, the branching ratios $\Gamma(I^+_\mathrm{b} \to I^+_\mathrm{b} - 2)$ / $\Gamma(I^+_\mathrm{b} \to \gamma)$ / $\Gamma(I^+_\mathrm{b} \to \mathrm{g})$ are known [14,15], which allow a determination of the B(E2, $I^+_\mathrm{b} \to \mathrm{b, g}, \gamma$) values as a function of the intrinsic quadrupole moment Q^b_0 assuming rotational B(E2) values for the in-band transitions of the b-band. As the B(E2, $I^+_\mathrm{b} \to \mathrm{g}, \gamma$) values for the 4^+_b and 6^+_b states follow generalized Alaga relations which allow for $\Delta K = 2$ mixing between the b- and the γ-band and $\Delta K = 0$ mixing between the b- and the g-band [16,25], respectively (for the $\Delta K = 0$ mixing see (4-219) and (4-220) of [1]), we used these relations to express all other inter-band matrix elements between the b-band and the γ- and g-band as a function of the intrinsic quadrupole moment Q^b_0. Thus the only free parameters in the COULEX calculations of the population of the b-band are Q^b_0 and the relative sign of the spin independent matrix elements $\langle K_\mathrm{b} = 0|\mathcal{M}(\mathrm{E2}, \nu = 0)|K_\mathrm{g} = 0\rangle$ and $\langle K_\mathrm{b} = 0|\mathcal{M}(\mathrm{E2}, \nu = 2)|K_\gamma = 2\rangle$, which plays an important role as the two excitation paths are strongly interfering. Choosing the same relative sign, a quadrupole moment of $Q^\mathrm{b}_0 = (7.5 \pm 0.5)$ eb (to be compared to the average intrinsic quadrupole moment of the g- and γ-band of $Q_0 \approx 8.0$ eb) reproduce the observed excitation probabilities of the 0^+_b and the 4^+_b states, as well as the weak excitation of the 2^+_b state very well. An opposite sign of the two matrix elements is ruled out, since it would result in a strong excitation of the 2^+_b state with $P(2^+_\mathrm{b})$ / $P(0^+_\mathrm{b}) \approx 2$, which is in disagreement with the observation. It should also be noted, that a calculation without any band mixing cannot reproduce the three excitation probabilities simultaneously. Our result for the mixing amplitude between the b- and γ-band is $|\langle\epsilon_2\rangle| \approx 6\times10^{-4}$, which is as expected in good agreement with the value extracted by Günther et al. [16], who however had to *assume* $Q^\mathrm{b}_0 = Q^\mathrm{g}_0$. The absolute E2 transition strengths between the 0^+_b band head and the g- and γ-band resulting from our Coulomb excitation analysis of the excitation probabilities of the b-band states are

$$\mathrm{B(E2}, 2^+_\mathrm{g} \to 0^+_\mathrm{b}) = (4.4 \pm 0.6)\ \mathrm{e^2fm^4}\ \text{and}$$
$$\mathrm{B(E2}, 2^+_\gamma \to 0^+_\mathrm{b}) = (30.4 \pm 4.6)\ \mathrm{e^2fm^4}.$$

Note that the given errors comprise all uncertainties of the input values to the analysis but do not include possible systematic uncertainties due to the use of the generalized Alaga relations. This is in contrast to the determination of the B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value, where systematic uncertainties are included in the error given.

4 Discussion

While the absolute B(E2) values connecting the 0^+_b band head to the g- and γ-band have been measured for the first time in the present experiment, the determination of the E2 transition strength between the $K^\pi = 4^+$ band head at 2056 keV and the 2^+ state of the γ-band has been attempted before. Employing the GRID lifetime technique, Börner et al. [4] could derive an upper and lower limit of the corresponding B(E2) value of 140 $\mathrm{e^2fm^4} \leq$ B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma) \leq 410$ $\mathrm{e^2fm^4}$. Moreover, using inelastic α-scattering data, Neu and Hoyer [26] deduced with the aid of several model assumptions an isoscalar mass transition moment for the $4^+_{\gamma\gamma} \to 2^+_\gamma$ transition, which they converted into an electromagnetic transition moment of B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma) = 320$ $^2\mathrm{fm}^4$. In view of their uncertainties and model dependencies both measurements mainly provided evidence that the E2 transition is enhanced compared to an incoherent two-quasiparticle excitation. More recently, however, a B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) value of (390 ± 90) $\mathrm{e^2fm^4}$ has been derived by Oshima el al. [27] from a Coulomb excitation measurement with ^{74}Ge projectiles, which can be directly compared to the present value of (600 ± 130) $\mathrm{e^2fm^4}$ as the same technique is used. Although both results seem to be consistent within errors, the difference is nevertheless puzzling as the error of our value is dominated by systematic errors caused by uncertainties in the branching ratio of the $4^+_{\gamma\gamma} \to 2^+_\gamma$ transition and the input values into the COULEX program (in particular of the assumption for $Q^{\gamma\gamma}_o$), which should effect both results in the same direction. Unfortunately, the information given in ref. [27] is not sufficient to investigate possible causes of this difference, nor is there any discussion on the ingredients to their error estimate. In the following discussion, we shall nevertheless use an average value of the two results but retain the error of $\pm$ 130 $\mathrm{e^2fm^4}$ because of its systematic character, i.e. we shall adopt B(E2, $4^+_{\gamma\gamma} \to 2^+_\gamma$) $= (500 \pm 130)$ $\mathrm{e^2fm^4}$; note, that the error also includes the upper limits of the previous estimates.

The experimentally known B(E2, $0^+_\mathrm{g} \to 2^+$) values for inter-band transitions in ^{168}Er are compiled in the upper part of table 1 together with the $0^+_\mathrm{g} \to 2^+_\mathrm{g}$ in-band E2 transition strength. The B(E2) values are taken from the present work or the literature. The B(E2, $0^+_\mathrm{g} \to 2^+_\mathrm{b}$) value was calculated from the B(E2, $0^+_\mathrm{b} \to 2^+_\mathrm{g}$) value using the

Table 1. Experimental results for the E2 collectivity built on the 0^+_g and 2^+_γ state of ^{168}Er

$I_1 \rightarrow I_2$	E_2 [keV]	B(E2, $I_1 \rightarrow I_2$) [e^{2}fm^4]	B(E2, $I_1 \rightarrow I_2$) [s.p.u.]^a
$0^+_\mathrm{g} \rightarrow 2^+_\mathrm{g}$	80	57000±1700^b	207±6
$0^+_\mathrm{g} \rightarrow 2^+_\gamma$	821	1320±50^b	4.8±0.2
$0^+_\mathrm{g} \rightarrow 2^+_\mathrm{b}$	1276	16±2^c	0.06±0.01
$0^+_\mathrm{g} \rightarrow 2^+_{K=0}$	≤ 2000	< 50	< 0.2
$2^+_\gamma \rightarrow 4^+_\gamma$	995	16000±1000^d	58±4
$2^+_\gamma \rightarrow 4^+_{\gamma\gamma}$	2056	900±230^e	3.3±0.8
$2^+_\gamma \rightarrow 0^+_\mathrm{b}$	1217	30.4±4.6	0.11±0.02

^{a} 1 s.p.u = 275 e^{2}fm^4
^{b} ref. [28]
^{c} calculated from B(E2,$2_\mathrm{g} \rightarrow 0_\mathrm{b}$) (see text)
^{d} ref. [23]
^{e} average value from ref. [27] and present work

generalized Alaga rule with parameters determined in the present Coulomb excitation analysis. The B(E2) values are also given in single particle units (s.p.u.) defined by $\mathrm{B_{s.p.u.}(E2)} = (5/4\pi)(3/5)^2(1.2A^{1/3})^4$ e^{2}fm^4 = 275 e^{2}fm^4 ≡ 5 $\mathrm{B_{W.u.}}$(E2), $\mathrm{B_{W.u.}}$(E2) being the usual Weisskopf unit. The table displays the typical hierarchy of the electric quadrupole strength between the ground state and the low-lying 2^+ states for well deformed even-even nuclei. The dominating E2 transition rate to the first excited 2^+_g state of more than 200 s.p.u. reflects the strong coherence of the many single particle excitations contributing to the 2^+_g state and is - together with the high excitation energies and the comparatively small E2 transition strengths to the other 2^+ states - the main key to the robustness of the rotational model description of the band structure of these nuclei. The strongest E2 inter-band transition strength connects the ground state to the $K^\pi = 2^+_\gamma$ state at 821 keV. Although the transition carries only 4.8 s.p.u., that is 2 % of the E2 strength of the $0^+_\mathrm{g} \rightarrow 2^+_\mathrm{g}$ transition, it is still enhanced compared to the E2 strength of typical non-collective two-quasiparticle excitations, which seem to be even less than 1 s.p.u. Together with its excitation energy being considerably lower than a typical quasiparticle excitation ($2\Delta \sim 1.8$ MeV) the enhancement indicates a moderately collective origin of the 2^+_γ state and its interpretation as the one-phonon γ-vibrational excitation of the 0^+_g state is suggestive. However, from the B(E2) values given in table 1 it is obvious that none of the excited $K^\pi = 0^+$ bands below ∼ 2 MeV can be interpreted as being built on the one-phonon β-vibrational excitation; the corresponding inter-band transitions between the excited $K = 0^+$ bands and the g-band do not exceed a few tenth of a s.p.u. This is in particular true for the lowest excited $K = 0^+_\mathrm{b}$ band starting at 1217 keV, for which an inter-band B(E2, $0^+_\mathrm{g} \rightarrow 2^+_\mathrm{b}$) value of (0.06 ± 0.01) s.p.u. has been deduced from the present data.

The stiffness of the heavy Er isotopes against β-vibrations as compared to their softness with regard to vibrations in the γ-direction has been explained in terms of the availability or non-availability of near lying single-particle orbits in these deformed nuclei, which can couple via the Y_{20} and $Y_{2\pm2}$ operators associated with the β- and γ-vibrations, respectively [1,25]. Microscopic calculations within e.g. the quasi-particle-phonon nuclear model (QPNM) by Soloviev et al. [29] also show that the low lying $K^\pi = 0^+$ states in ^{168}Er are non-collective excitations, while the 2^+_γ state is a coherent superposition of several two-quasiparticle excitations; the calculated B(E2) values of 4.4 s.p.u. for the $0^+_\mathrm{g} \rightarrow 2^+_\gamma$ and of 0.07 s.p.u. for the $0^+_\mathrm{g} \rightarrow 2^+_\mathrm{b}$ transition are in good agreement with the experimental values of 4.8 s.p.u. and 0.06 s.p.u., respectively.

The in-band E2 properties of the low spin members of the three bands built on the $0^+_\mathrm{g}, 2^+_\gamma$ and 0^+_b states, respectively, are well described by the rotational model with intrinsic quadrupole moments of $Q^\mathrm{g}_0 = (8.1 \pm 0.3)$ eb [23] $Q^\gamma_0 = (7.8 \pm 0.4)$ eb [23] and $Q^\mathrm{b}_0 = (7.5 \pm 0.5)$ eb (present experiment). They agree within their experimental errors and thus support the assumption that the equilibrium deformation of ^{168}Er in the two excited bands is identical to that of the ground state. Note, however, that the E2 collectivity connected with the intrinsic excitation is very weak compared to the in-band E2 strength, which makes the inter-band transitions rather sensitive to band mixing effects. This is borne out e.g. by the necessity to use generalized Alaga relations to describe properly the transitions between the three bands. For the transitions between the lowest spin members of each band, however, these effects change the B(E2) values by less than 20 % and thus do not influence the present discussion.

The experimentally known E2 transition strengths between the 2^+_γ state and higher lying states are given in the lower part of table 1. As to be expected, the in-band transition is again by far dominating, however, the B(E2) value for the inter-band transition to the $K^\pi = 4^+$ state at 2056 keV amounts to 6 % of the B(E2, $2^+_\gamma \rightarrow 4^+_\gamma$) value and is thus similarly enhanced as the $0^+_\mathrm{g} \rightarrow 2^+_\gamma$ transition. It has therefore been tempting for many authors to interpret this state as a manifestation of the $K^\pi = 4^+$ $\gamma\gamma$-two-phonon state. In the left part of table 2 the predictions of several models for the excitation energy and E2 collectivity of the $K = 4^+$ $\gamma\gamma$-vibrational state are given together with the corresponding experimental values for the 4^+ state at 2056 keV. The experimental B(E2) ratio, $B(4^+_{\gamma\gamma})$ = B(E2, $4_{\gamma\gamma} \rightarrow 2^+_\gamma$) / B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{g}$) = (1.9 ± 0.4), exhaust only 70 % of the strength expected for a harmonic vibration, however, the deviation of the experimental excitation energy ratio $R(4^+_{\gamma\gamma}) = E(4^+_{\gamma\gamma})/E(2^+_\gamma) = 2.5$ from the harmonic value of 2.0 indicates that the γ-vibration is not harmonic but subject to a large anharmonicity . Already in [9,25] it was pointed out that this could be readily explained if the potential energy surface would have a shallow minimum at a non-zero value of the γ-deformation, shallow compared to the potential energy surface at $\gamma = 0°$ and to the zero point energy of the γ-vibration. In fact, calculations by Matsuo et al. [10] using a microscopic Hamiltonian together with the self-consistent collective-coordinate method (SCCM) do predict a potential energy surface with a minimum at $\gamma \approx 10°$ and one- and

Table 2. Various model predictions for the $K^\pi = 4^+$ and 0^+ $\gamma\gamma$-two-phonon vibration

Model	$K^\pi = 4^+$		$K^\pi = 0^+$	
	$R(4^+_{\gamma\gamma})^a$	$B(4^+_{\gamma\gamma})^b$	$R(0^+_{\gamma\gamma})^a$	$B(0^+_{\gamma\gamma})^b$
Harm. Vib.	2.0	2.78	2.0	5.0
SCCM [10]	2.5	1.9	2.7	1.25
IBM–sdg [30]	2.5	1.4	1.5	1.8
QPNM [29]	2.5	0.73	strongly fragmented	
MPM [31]	2.8	0.53	3.5	0.1
DDM [13]			1.2	10.3
Experiment	2.5	1.9 ± 0.5	$(1.48)^c$	$(0.6 \pm 0.1)^c$

[a] $R(I^+_{\gamma\gamma}) = E(I^+_{\gamma\gamma})/E(2^+_\gamma)$
[b] $B(I^+_{\gamma\gamma}) = \mathrm{B(E2,}I^+_{\gamma\gamma} \rightarrow 2^+_\gamma) / \mathrm{B(E2,}2^+_\gamma \rightarrow 0^+_\mathrm{g})$
[c] experimental values for the $K^\pi = 0^+_\mathrm{b}$ state at 1217 keV

two-phonon states with $R(4^+_{\gamma\gamma})$ and $B(4^+_{\gamma\gamma})$ ratios in perfect agreement with the present data (see table 2). Moreover, a finite γ value results also from a multiple Coulomb excitation experiment [23], in which $\gamma \approx 8°$ was extracted.

While the extended version of the Interacting Boson Model (IBM-sdg) [30] is in moderate accord with the experimental values, the QPNM [29] and the Multi-Phonon Method [31] are both underestimating the $B(4_{\gamma\gamma})$ ratio by a factor of 3 and 4, respectively. The short-coming of the QPNM calculation can be traced to the fact that only about 30% of the calculated wave-function of the 4^+ state is due to the $\gamma\gamma$ component while the rest is predicted to be made up by two-quasiparticle excitations of the ^{168}Er ground state, which add up to form a $K^\pi = 4^+$ hexadecapole phonon. This seems not only to be at variance with the experimental B(E2) ratio but also with the B(E4, $0^+_\mathrm{g} \rightarrow 4^+_{\gamma\gamma}$) value deduced from inelastic α scattering [26], which is more than a factor of 4 smaller than the QPNM prediction.

Using the fact that two-quasiparticle (one-phonon) states can be populated in single-nucleon transfer reactions while two-phonon states, involving four quasiparticles, usually cannot, Burke [32] has shown that many of the previously proposed candidates for $\gamma\gamma$-two-phonon states in deformed nuclei are actually two-quasiparticle instead of two-phonon states. For ^{168}Er none of the transfer reactions studied so far ($(d,p), (t,d), (\alpha,t), (t,p), (p,t)$) [33–35] has resulted in a measurable population of the low spin members of the $K^\pi = 4^+$ band at 2056 keV. This is consistent with - but not necessarily conclusive for - the assignment of this band as the $K^\pi = 4^+$ $\gamma\gamma$-two-phonon band. Note, moreover, that also the small branching ratio of the decay of the $4^+_{\gamma\gamma}$ state to the 2^+_g state determined in the present measurement to be B(E2, $4^+_{\gamma\gamma} \rightarrow 2^+_\mathrm{g}$) / B(E2, $4^+_{\gamma\gamma} \rightarrow 2^+_\gamma$) $< 7 \times 10^{-3}$, is consistent with but not conclusive for the two-phonon assignment. Despite the softening of the K selection rule by the observed band mixing one still expects a considerable smaller than observed branching ratio from the K selection rule alone; the sensitivity necessary to see an effect due to the phonon number selection rule has not been reached yet.

Thus, all the experimentally known properties of the 4^+ state at 2056 keV are consistently described by an anharmonic $K^\pi = 4^+$ $\gamma\gamma$-two-phonon excitation. Therefore the question arises where the $K = 0^+$ $\gamma\gamma$ strength is located. According to the SCC Model [10] this state is expected around 2.2 MeV. However, no $K^\pi = 0^+$ bands are known in this excitation energy region and possible candidates decaying to the $K^\pi = 2^+$ one-phonon band have not been observed in the present experiment (see also Fig. 2) although we are in fact quite sensitive to these transitions. Of the three known $K^\pi = 0^+$ bands in ^{168}Er only the b-band at 1276 keV was strongly enough excited to allow for a study of the absolute E2 collectivity between the γ- and the b-band. The deduced B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{b}$) of 30.4 ± 4.6 e^{2}fm^4 amounts to only 12 % of the B(E2, $2^+_\gamma \rightarrow 0^+_\mathrm{g}$) value (see table 1), indicating that at most 10 % of the wave function of the 0^+_b state can be attributed to a $\gamma\gamma$-two-phonon state. This is in clear contradiction to the claim of [11,12], who interpret the comparatively pronounced decays of the b-band to the γ-band as evidence for a significant $\gamma\gamma$-two-phonon amplitude in the b-band. Their argument is based on the R' ratio, defined by R' = B(E2, $0^+_\mathrm{b} \rightarrow 2^+_\gamma$) / B(E2, $0^+_\mathrm{b} \rightarrow 2^+_\mathrm{g}$) and which they estimated from the branching ratios of the higher spin members (I^π_b) of the b-band to $R' = 110$ ($I^\pi_\mathrm{b} = 6^+$) and $R' = 56$ ($I^\pi_\mathrm{b} = 4^+$) without taking the band mixing into account. As pointed out already by Günther et al. [16] this procedure results in very unreliable R' ratios. In fact, we obtain $R' = 6.9 \pm 1.4$ if we use our measured B(E2, $0^+_\mathrm{b} \rightarrow 2^+_\gamma$) and B(E2, $0^+_\mathrm{b} \rightarrow 2^+_\mathrm{g}$) values, in agreement with the estimate of Günther. It should moreover be noted that the more telling assertion with respect to the $\gamma\gamma$-two-phonon character of the 0^+_b state is the direct comparison of the $0^+_\mathrm{b} \rightarrow 2^+_\gamma$ transition strength with the collectivity of the γ-phonon rather than the $0^+_\mathrm{b} \rightarrow 2^+_\mathrm{g}$ collectivity. The R' ratio being in favour of the transition to the γ-band is not due to an enhanced $0^+_\mathrm{b} \rightarrow 2^+_\gamma$ E2 strength but to a rather unusual low B(E2, $0^+_\mathrm{b} \rightarrow 2^+_\mathrm{g}$) value.

On the right side of table 2 the predictions of several models for the $K^\pi = 0^+$ $\gamma\gamma$-two-phonon state are given in terms of the two ratios $R(0^+_{\gamma\gamma})$ and $B(0^+_{\gamma\gamma})$. It is obvious that none of the $0^+_{\gamma\gamma}$ model states can be related to the 0^+_b state; they predict either too high excitation energies or too large B(E2) values. The only one of these models that is able to quantitatively reproduce the properties of the b-band, in particular also the branching to the γ-band, is the QPNM [29]. In this picture, however, the $K^\pi = 0^+_\mathrm{b}$ state is an incoherent superposition of several two-quasiparticle excitations, the $K^\pi = 0^+_{\gamma\gamma}$ strength being shifted above 2.3 MeV and strongly fragmented.

In summary, the $K^\pi = 2^+$ level at 821 keV and the $K^\pi = 4^+$ state at 2056 keV in ^{168}Er can be considered as a realization of the one- and two-γ-phonon excitation, respectively, of the strongly β- and slightly γ-deformed ground state. The situation is thus similar to that encountered in ^{166}Er, although in the latter nucleus the $K = 4^+$ $\gamma\gamma$-two-phonon strength seems to be somewhat more fragmented. On the other hand a $K^\pi = 0^+$ state could be identified in ^{166}Er [5–7], which carries a considerable part of the $K^\pi = 0^+$ $\gamma\gamma$-two-phonon strength, while a corresponding state has not yet been found in ^{168}Er. It remains to be seen if such a state can be isolated at all in this nucleus. But even so, in view of the great effort of many research groups over the last decades to identify two-quadrupole-phonon states in strongly deformed nuclei and the still meager number of convincing examples, the question arises how useful the phonon picture is in describing the low-lying intrinsic excitations of a deformed nucleus. A purely macroscopic picture of a deformed nucleus performing surface vibrations around an equilibrium deformation does not seem to have any predictive power. As discussed above, even the presence or absence of a one-phonon β- or γ-vibrational mode is intimately connected with the available single particle orbits close to the Fermi surface. Thus at least a microscopic description of the macroscopic Hamiltonian is required to understand the excitation energy and collectivity of the lowest vibrational states, that is of the one-phonon vibrations. However, these calculations also show, in agreement with experimental findings, that the collectivity of these phonons is small and rather fragile as they are composed out of a few coherent quasiparticle excitations only. Thus in many nuclei already the two-phonon amplitude is destroyed by being spread over several other quasiparticle excitations, making the occurrence of rather pure two-phonon surface vibrations in deformed nuclei an exception rather than a rule. This is in contrast to the rotational motion of the deformed nucleus, which provides a remarkable good description of the bands built on the intrinsic excitations, and e.g. in contrast to the Giant Resonance vibrations where the existence of two-phonon and possibly even higher excitations has been demonstrated [36]. In both cases the collectivity as measured by the B(Eλ) values is obviously pronounced enough to cope with the changes of the internal single particle structure making up the rotation and vibration. Surface vibrational modes on the other hand, seem to be of only limited use in describing the intrinsic excitations of deformed nuclei.

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